Date ....

Permutation & Combination

Permutation → Arrangement → order matter

Combination → Selection → order Doesn't matter

- Factorial (!) → $4! = 4\times3\times2\times1$
  → $3! = 3\times2\times1$
  → $2! = 2\times1$
  → $5! = 5\times3\times2\times4\times1$

$1! = 1$

$0! = 1$

eg:- $7! = 7\times6\times5\times4\times3\times2\times1 = 5040$

$6! = 6\times5\times4\times3\times2\times1 = 720$

$8! = 8\times7\times6\times5\times4\times3\times2\times1 = 40320$

$\Rightarrow 7! - 6!$

$= 5040 - 720 = 4320$

$\Rightarrow 7! - 6!$

$= 7\times6! - 6! = 6!(7-1) = 6.6! = 4320$

$\Rightarrow \frac{7! - 5!}{2} = \frac{5040 - 120}{2} = 2460$

$= \frac{5![7\times6-1]}{2} = \frac{120\times41}{2} = 2460$

$$\Rightarrow \frac{1}{7!} + \frac{1}{8!} = \frac{8+1}{8!} = \frac{9}{8!}$$

$$\Rightarrow \frac{1}{8!} + \frac{1}{9!} + \frac{1}{10!} = \frac{10\times 9+10+1}{10!} = \frac{101}{10!}$$

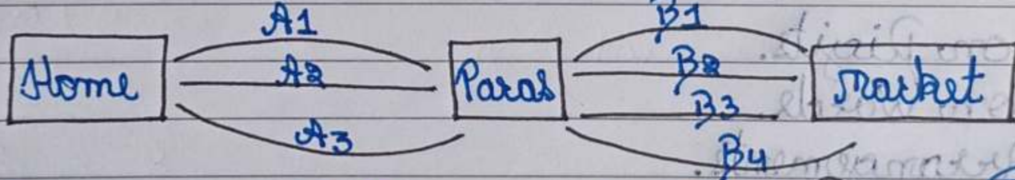
$$\Rightarrow^{*}\ n! - (n-1)! = n(n-1)! - (n-1)! = (n-1)!(n-1)$$

$$\Rightarrow^{*}\ \frac{1}{n!} + \frac{1}{(n+1)!} = \frac{n+1+1}{(n+1)!} = \frac{n+2}{(n+1)!}$$

Addition → OR

Multiplication → AND

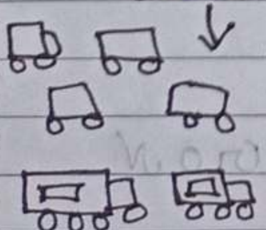
eg:-



A1 B1	A2 B1	A3 B1
A1 B2	A2 B2	A3 B2
A1 B3	A2 B3	A3 B3
A1 B4	A2 B4	A3 B4

$\Rightarrow$ 3×4 = 12 Way

eg:- Paras → Home



4 Autos

2 Buses

$\Rightarrow 4+2 = 6$ Way

$\Rightarrow$ Simultaneously not Possible

Spiral

Date ....

eg:- $\square\square\square$ A B C

$3 \times 2 \times 1 = 6$

$\Rightarrow 3!$

eg:- $\square\square\square$ 5 Persons.

$5 \times 4 \times 3 = 60$

Permutations

$\boxed{{}^{n}P_{r} \Rightarrow \frac{n!}{(n-r)!}}$ $\boxed{{}^{n}P_{n} \Rightarrow n!}$

(i) r Factors.
(ii) $n \geq r$.
(iii) Repetition not allowed.

Never → Total - Always come

Permutation [order matter]

(i) Problems on Digits.
(ii) Problems on Words.
(iii) Sitting Arrangements.

Eg:- m Imp HEXAGON

Vowels → E A O [girls]
consonants → H X G N [Boys]

(i) when there is no restriction :- $7!$
(ii) H first Place. :- $6!$
(iii) starting with H end with N :- $5!$
(iv) Vowels occur Together :- $5! \times 3!$

$\boxed{E\,A\,O}$ H X G N HEXAGON

Spiral

Date ....

(v) Vowels Never occur Together:-

Total - Always come.

$7! - 5!\,3!$

(vi) No Two Vowels occur Together:-

E A O HEXAGON

_ H _ G _ N _ X _

$\rightarrow 4!\ {}^5P_3$

HEXAGON

→ Relative Position of Vowels & consonants remain same.

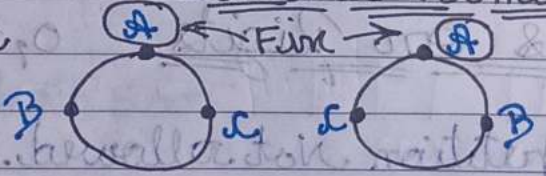
⇒ H X G N = 4!

E A O = 3!

⇒ Vowel! consonants!

circular Permutation

eg:- A B C



A B C
A C B
B A C
B C A
C A B
C B A

$\left.\right\} 2!$

$\boxed{n\text{things} \rightarrow (n-1)!}$

- case of Necklese. $\rightarrow \frac{(n-1)!}{2}$
- Their is no change in clockwise & anticlock wise Direction.
- They Don't have same neighbour.

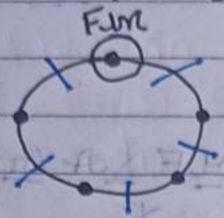
Spiral

Date ....

• Restriction :- No Two. circular Table.

eg :- 5 Boys 4 girls

No Two girls sit Together.



$4! \; {}^5P_4$

⇒ Restricted should be Treated as liner.

* Books → subject → Different
* Balls → colour →
  - • P → Identical.
  - • C → different.

⇒ never = Total - Always come.

$= n! - (n-1)!\,2!$ $\quad n-2+1$

$= (n-1)!\,[n-2]$ $\quad = n-1$

Problems on Digits :

0, 1, 2 - - - - - 9

10 digits

• Selected Digits → Repetition Not allowed.

• Interval → 100 to 1000

Selected digits → Repetition Not Allowed.

→ Repetition Allowed.

eg :- four digit No's 2, 3, 5, 7, 8, 9

(i) [ ][ ][ ][ ]

$6 \times 5 \times 4 \times 3 \Rightarrow 360$

(ii) 3 comes at unit Place:-

$\boxed{}\boxed{}\boxed{}\boxed{3}$

$5 \times 4 \times 3 \Rightarrow 60$

(iii) No's begining with 5 ending with 9:-

$\boxed{5}\boxed{}\boxed{}\boxed{9}$

$4 \times 3 \Rightarrow 12$

(iv) Even No's:-

$\boxed{}\boxed{}\boxed{}\boxed{}$

$5 \times 4 \times 3 \underset{2}{\longleftrightarrow} \Rightarrow 120.$

(v) odd No's:-

$\boxed{}\boxed{}\boxed{}\boxed{3}$

$5 \times 4 \times 3 \times 4 = 240.$

(vi) Greater than 5000

$\boxed{8}\boxed{}\boxed{}\boxed{}$

$4 \times 5 \times 4 \times 3 \Rightarrow 240.$

(vii) less than 7000

$\boxed{}\boxed{}\boxed{}\boxed{}$

$3 \times 5 \times 4 \times 3 = 180$

(viii) lying b/w 3000 to 7000

$\boxed{}\boxed{}\boxed{}\boxed{}$

 $2 \times 5 \times 4 \times 3 \Rightarrow 120$

Date ....

(Imp)

(ix) Divisible by 4:-

$\boxed{\;}\boxed{\;}\boxed{2}\boxed{8}$

$4 \times 3 \times 5 = 60$

(x) odd No's greater than 5000:-

$\boxed{9}\boxed{\;}\boxed{\;}\boxed{\;} \Rightarrow 48 + 108 = 156$

3. 4. 3. 3

(xi) Even No's less than 7000:-

2, 3 5

$\boxed{2}\boxed{\;}\boxed{\;}\boxed{8}$ $\quad$ $\boxed{5}\boxed{\;}\boxed{\;}\boxed{8}$ $\quad \Rightarrow 12 \times 48 = 576$

$4 \times 3 \qquad 2 \times 4 \times 3 \times 2$

(xii) No's greater than 2500:-

$\boxed{\;}\boxed{\;}\boxed{\;}\boxed{\;}$

25000 - 3000 greater than 3000

$\boxed{2}\boxed{9}\boxed{\;}\boxed{\;} + \boxed{9}\boxed{\;}\boxed{\;}\boxed{\;} \Rightarrow 48 + 300 = 348$

$4 \times 4 \times 3 \qquad 5 \times 5 \times 4 \times 3$

Combination

↓

order Doesn't matter

2 Persons select → A B C D E

$^3C_2 \Rightarrow 3 \qquad ^4C_2 = 6 \qquad ^5C_2 = 10$

$$\boxed{^nC_r \Rightarrow \frac{n!}{(n-r)!\,r!}} \Rightarrow \boxed{^nC_r \Rightarrow \frac{^nP_r}{r!}}$$

Imp

eg:- $^{10}C_4 \Rightarrow \frac{10\times \cancel{9}^{3}\times \cancel{8}\times 7}{\cancel{4}\times\cancel{3}\times\cancel{2}\times 1} = 210$

$^5C_2 \Rightarrow \frac{5\times \cancel{4}^{2}}{\cancel{2}} = 10.$

$^5C_3 \Rightarrow \frac{5\times\cancel{4}\times\cancel{3}}{\cancel{3}\times\cancel{2}} = 10$

Imp rule $\boxed{^nC_r = {}^nC_{n-r}}$

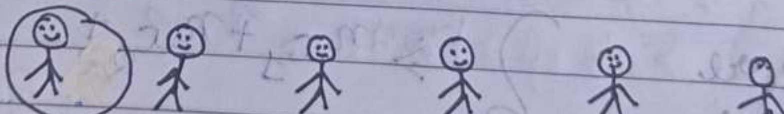
$\boxed{^nC_x = {}^nC_y}$ $x = y$
$x + y = n$

eg:- $^{14}C_3 = {}^{14}C_x \Rightarrow x = ?$ 3, 11

$\Rightarrow$ * $\boxed{^{n+1}C_r = {}^nC_r + {}^nC_{r-1}}$ * Property

eg:- 6 Players. 3 Players select.

$^6C_3 = \frac{6\times5\times4}{3\times2\times1} = 20$



I E

$^5C_2 + {}^5C_3$

10 + 10

$\Rightarrow 20$

eg:- $^{14}C_3 + {}^{14}C_4 = {}^{15}C_x \Rightarrow x = ?$

4 or 11 Ans

Date ....

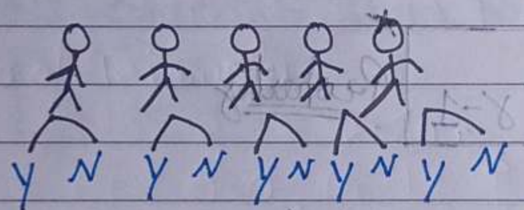
$$\Rightarrow \boxed{{}^{n}C_{r} + 2\,{}^{n}C_{r-1} + {}^{n}C_{r-2} = {}^{n+2}C_{r}}$$

eg:- 5 friends invite for a Party
one or more friends

$${}^{5}C_{1} + {}^{5}C_{2} + {}^{5}C_{3} + {}^{5}C_{4} + {}^{5}C_{5}$$

$$5 + 10 + 10 + 5 + 1 = 31$$

$\Rightarrow$ atleast one $\rightarrow$ Total - No one or more.



$$2 \times 2 \times 2 \times 2 \times 2 - 1 = 31$$

$$\Rightarrow 2^5 - 1 = 31$$

\# One or more
\# Atleast one
\# Some or All
\# Some of atlest one

$\rightarrow {}^{n}C_{1} + {}^{n}C_{2} + \ldots\ldots {}^{n}C_{n}$

$\rightarrow 2^n - 1$

$\rightarrow \sum_{r=1}^{n} {}^{n}C_{r} = 2^n - 1$

Spiral

Date 50

eg:- 5 Players 3 Player select

one particular Player always come.

$^4C_2$

one particular Player Never come.

$^4C_3$

eg:- M.Imp

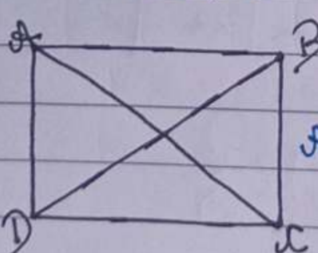
question

O → A: 8 questions; O → B: 10 questions

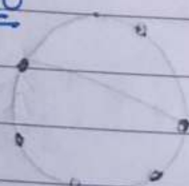
atleast 3 from each

$$^8C_3 \times {}^{10}C_6 + {}^8C_5 + {}^8C_5 \times {}^{10}C_4 + {}^8C_6\,{}^{10}C_3$$

$$\Rightarrow \frac{8\times7\times6}{3\times2} \times \frac{10\times9\times8\times7}{4\times3\times2} + \frac{8\times7\times6\times5}{4\times3\times2} \times \frac{10\times9\times8\times7\times6}{120} + \frac{8\times7\times6}{3\times2} \times \frac{10\times9\times8\times7}{4\times3\times2} + \frac{8\times7}{2} \times \frac{10\times9\times8}{3\times2} \Rightarrow 44520$$



A B C D 4 Points straight line



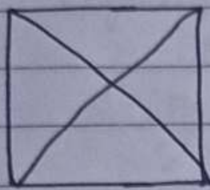
AB BC CD
AC BD
AD

combination

⇒ geometrical Problems

⇒ select, choose, choice, committee group, team, Party, club.

Date ....

- No of straight lines $\rightarrow {}^nC_2$
- No of Triangles $\Rightarrow {}^nC_3$
- No.  of Diagonals $\rightarrow {}^nC_2 - n$

$\Rightarrow$ n Points m Points collinear

No of straight lines $\rightarrow$ $\boxed{{}^nC_2 - {}^mC_2 + 1}$

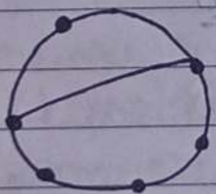
eg:- 10 Points 7 Points collinear.

No of Triangles

$\Rightarrow {}^{10}C_3 - {}^7C_2$

n Points m Points

No. of Triangles :- ${}^nC_3 - {}^mC_3$



$\rightarrow$ No of chords $\Rightarrow {}^nC_2$

$\rightarrow$ No of triangle $\Rightarrow {}^nC_3$

$\rightarrow$ No of quadrilateral $\Rightarrow {}^nC_4$

journey

one sided journey $\rightarrow {}^nC_2$

Two sided journey $\rightarrow {}^nP_2$

⇒ n things m things arrange
↳ one particular things always occurs.

5 things 3 things arrange
↳ one — Always

$$\Rightarrow \cancel{O} \quad O \quad O \quad O \quad O$$

$$^{4}C_{2} \; 3!$$

⇒ 5 things 3 things arrange
↳ one particular thing Never occur.

$$\cancel{O} \quad O \quad O \quad O \quad O$$

$$\Rightarrow {}^{4}C_{3} = {}^{4}C_{3}\, 3!$$

⇒ n things r things arrange.
↳ one Always.

$$\Rightarrow \boxed{r \cdot {}^{n-1}P_{r-1}} \longleftrightarrow \boxed{{}^{n-1}C_{r-1} \cdot r!}$$

Permutation Formula

combination Formula

Date ....

Rank Find :-

$\Rightarrow 720 = 6!$

Short cut

Z E N I T H

6 1 9 3 5 2

$5 \times 5! + 0 \times 4! + 2 \times 3! + 1 \times 2! + 1 \times 1! + 0 \times 0! + 1$

$600 + 12 + 2 + 1 + 1 = 616$

eg :- J U L A D

4 5 3 1 2

$3 \times 4! + 3 \times 3! + 2 \times 2! + 0 \times 1! + 0 \times 0! + 1$

$\Rightarrow 72 + 18 + 4 + 1 = 95$

G O O G L E

2 4 4 2 3 1

$\frac{1}{2!2!} \times 5! + \frac{3}{2!} \times 4! + \frac{3}{0!} \times 3! + \frac{1}{0!} \times 2! + \frac{1}{0!} \times 1! + \frac{0}{0!} \times 0! + 1$

$\Rightarrow \boxed{88}$

$\Rightarrow$ n things $\rightarrow$ r things $\rightarrow$ Arrange / $\rightarrow$ Select

i) 2 a like 2 a like

ii) 2 a like 2 different

iii) 4 are different

MIND MAP

RITU JINDAL

PERMUTATION AND COMBINATION ⇒ factorial

Fundamental Principle of Counting

- Addition Theorem ⇓ $m+n$ ways — Or
- Multiplication Theorem ⇓ $m \times n$ ways — And

Main Problems

⇓

- # Word Problems
- # Sitting Arrangement
- # Problems on Digit

Permutation

⇓ Arrangement = Order matter

Types of Questions

- n things r places $^nP_r = \frac{n!}{(n-r)!}$ $n \geq r$ Q.N.A
- n things n places $^nP_n = \frac{n!}{(n-n)!} = n!$
- n things p alike, q alike.. $\Rightarrow \frac{n!}{p!\,q!\cdots}$
- Repetition allowed n^r To Compulsory

Restricted Permutation

- n things r at a time. One Particular thing always occur ⇓ $r\,{}^{n-1}P_{r-1}$
- n things r at a time. One particular thing Never Occur ⇓ $^{n-1}P_r$
- n things taken all at a time m specified things always Come together ⇓ $(n-m+1)!\,m!$
- n things taken all at a time m specified things Never Come together ⇓ $n! - m!(n-m+1)!$

Circular Permutation ⇒ $(n-1)!$

- No change in Clockwise and anticlockwise Direction ⇒ They Don't have Same Neighbourhood in Any two occassion ⇒ Necklace or Garland ⇓ $(n-1)!$

MIND MAP

PERMUTATION AND COMBINATION

RITU JINDAL

COMBINATION

⇓

Selection or Order Doesn't matter

Types of Questions

- n things Taken r at a time: $^nC_r = \frac{n!}{r!(n-r)!} = \frac{^nP_r}{r!}$
- n things Taken all at a time: $^nC_n = \frac{n!}{n!(n-n)!} = 1$
- n things taken r at a time 1 Particular always occur: $^{n-1}C_{r-1}$
- n things Taken r at a time 1 Particular Never occur: $^{n-1}C_r$

Some Important Results

- $^nC_r = {^nC_{n-r}}$
- $^nC_x = {^nC_y}$, $x = y$ or
- $^nC_r + {^nC_{r-1}} = {^{n+1}}$
- $^nC_1 + {^nC_2} + \dots$ $\dots {^nC_n} =$

Geometrical Problems

- Straight Line
  - Simple ⇓ $^nC_2$
  - Collinear ⇓ $^nC_2 - {^mC_2} + 1$
- Triangle
  - Simple ⇓ $^nC_3$
  - Collinear ⇓ $^nC_3 - {^mC_3}$
- Parallelogram ⇓ $^mC_2\,{^nC_2}$
- No. of Diagonals ⇓ $^nC_2 - n$ or $\frac{n(n-3)}{2}$
- No of Chords ⇓ $^nC_2$

Some Important Points

⇓

Selected digits given Repetition Not allowed

Balls → Colour
- Permutation → Identical
- Combination → Different

Books → Subject → Different